APPLICATION OF INFORMATION THEORY TO REACTOR PHYSICS

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KALPAKKAM
INDIA.

SCOPE OF THE TALK

• QUALITATIVE INTRODUCTION TO INFORMATION THEORY.
• CONSTRUCTION OF COVARIANCE MATRIX.
• REDUCTION OF SYSTEMATIC UNCERTAINTY.
• APPLICATION TO REACTOR PHYSICS.

INFORMATION AGE

• ABUNDANT INFORMATION.
• INTERNET.
• INFORMATION AND UNCERTAINTY.
• UNCERTAINTY = [1 / INFORMATION]

COMMUNICATION THEORY.

• INPUT SIGNAL
• OUTPUT SIGNAL
• FOR ZERO NOISE
• FOR NON ZERO NOISE

COMMUNICATION THEORY

• OUT SIGNAL = [IN SIGNAL] / [NOISE]
• LESS THE NOISE MORE THE OUT SIGNAL.
• NOISE LOSS OF INFORMATION.
• LOSS OF INFORMATION UNCERTAINTY.
QUALITY OF DATA
- Uncertainty depends on the quality of data.
- Assessment of quality.
- Covariance matrix.
- Generation of covariance matrix using quality data.

BASIC STATISTICS
- Mean = \( \mu = \langle x \rangle \)
- \( dx = x - \mu_x, \ dy = y - \mu_y \), then
- Variance = \( < dx \ dx > \)
- Std. dev = \( \text{SQRT} \ [\text{VARIANCE}] \)
- Covariance = \( < dx \ dy > \)
- \( \rho = \frac{< dx \ dy >}{\text{SD}[x] \ \text{SD}[y]} \)

COUNTING EXPERIMENT
Two Sources 1 and 2.
\[
\begin{align*}
N_1 &= G_1 - B = 900 - 700 = 200 \\
N_2 &= G_2 - B = 981 - 700 = 281 \\
SD(N_1) &= \text{SQRT}(900 + 700) = 40. \\
SD(N_2) &= \text{SQRT}(981 + 700) = 41. \\
RSD(N_1) &= \frac{SD(N_1)}{N_1} = 0.2 \\
RSD(N_2) &= \frac{SD(N_2)}{N_2} = 0.146.
\end{align*}
\]

ERROR (VARIANCE) MATRIX
\[
\begin{array}{cccc}
& \text{Var (G1, G2, B)} = \\
\text{900} & \text{981} & \\
\text{981} & \text{700} & \\
\end{array}
\]

COV. AND COR. MATRIX
\[
\begin{array}{cccc}
1600 & 700 & \\
1.0 & 0.427 & \\
700 & 1681 & \\
0.427 & 1.0 & \\
\end{array}
\]

TYPES OF ERROR
- Random error:
  - Stochastic fluctuations.
  - Has a second moment.
  - Uncorrelated.
- Systematic error:
  - Remains constant.
  - No second moment.
  - Correlated.
RANDOM ERROR
• REDUCED BY REPETITION.
• STATISTICAL TECHNIQUES.
• NO LIMITS FOR THE RANDOM ERROR.

NORMAL DISTRIBUTION

SYSTEMATIC ERROR
• CANNOT BE REDUCED BY REPETITION.
• NEED FOR LIMITS FOR SYSTEMATIC ERROR.
• MATHEMATICAL MODEL.

REQUIREMENTS OF CENTRAL LIMIT THEOREM
• VARIABLES SUMMED MUST BE INDEPENDENT.
• ALL VARIABLES MUST HAVE FINITE MEAN AND VARIANCE.
• NO VARIABLE CAN MAKE AN EXCESSIVELY LARGE CONTRIBUTION TO THE SUM.

SYSTEMATIC ERROR
• ENTROPY BASED APPROACH.
• $H = \int f(x) \log [f(x)] \, dx$.
• LARGER THE ENTROPY, GREATER THE UNCERTAINTY.
• MATHEMATICAL MODEL.
• DETERMINANT INEQUALITIES.

SOURCES OF DISCREPANCY IN REACTOR CALCULATIONS
• MODELLING.
• CALCULATIONAL METHODS.
• MONTE CARLO.
• TRANSPORT THEORY
• NEUTRON X-SECTIONS.
• FOCUS ON X-SECTIONS.
**NEUTRON X-SECTIONS**

**ABSOLUTE**

- REACTION RATE = C = N\(\sigma\)\(\phi\);
- \(\sigma = \{ C / N \ \phi \}\)

**RELATIVE**

- \(\sigma_i / \sigma_j = \{ C_i / C_j \} \{ N_j / N_i \}\)

**RATIO OF COUNT RATES**

- \(\sqrt{([RSD(N1)]^2 + [RSD(N2)]^2)} = 0.25\)
- \(\sqrt{([RSD(N1)]^2 + [RSD(N2)]^2 - CF)} = 0.19\)
- \(CF = 2 \rho \cdot RSD[N1] \cdot RSD[N2]\)

**RELATIVE X-SEC. MEASUREMENT**

- \(\sigma = \sigma (p_i, p_j, p_k, \ldots)\)
- \(M_\sigma = \text{REL. COV.MATRIX OF } \sigma\)
- \(M_p = \text{REL. COV.MATRIX OF } p\)
- \(M_p = B M_\sigma B^T \text{ LAW OF ERROR PROPAGATION.}\)
- INF. \(M_\sigma\) DEPENDS ON INF. \(M_p\)

**MODEL TO REDUCE SYSTEMATIC UNCERTAINTY.**

- UNCTY. \(M_\sigma\) DEPENDS ON UNCTY \(M_p\)
- MATHEMATICAL MODEL.
- DETERMINANT INEQUALITIES.
- REDUCE UNCTY. \(M_p\)

**PARAMETERS IN X-SEC MEASUREMENT**

- \(p_1\) C Count Rates,
- \(p_2\) \(\epsilon\) Efficiency of the detector,
- \(p_3\) Geometrical Area,
- \(p_4\) Half life,
- \(p_5\) Back scattering,

**DETERMINANT INEQUALITIES**

- SIGN OF THE DET.
  - \(\det M_p > 0\) FOR \(\rho = 0\).
  - \(\det M_p = 0\) FOR \(\rho \pm 1\).
  - \(\det M_p \geq 0\)

<table>
<thead>
<tr>
<th>(\sigma_X^2)</th>
<th>(\rho_{XY})</th>
<th>(\rho_{XZ})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_Y^2)</td>
<td>(\sigma_Y)</td>
<td>(\sigma_Z)</td>
</tr>
<tr>
<td>(\rho_{YZ})</td>
<td>(\sigma_Y)</td>
<td>(\sigma_Z)</td>
</tr>
<tr>
<td>(\rho_{ZY})</td>
<td>(\sigma_Y)</td>
<td>(\sigma_Z)</td>
</tr>
</tbody>
</table>
**ESTIMATION OF BOUNDS**
- LIMITS FOR SYSTEMATIC ERROR.
- Let \( q \) Constant Bias.
- \( \text{Det.} \rho(q) \geq 0 \).
- ALGORITHM.
- PROCESS HIGHER ORDER MATRICES.

<table>
<thead>
<tr>
<th>( \rho_{12}(q) )</th>
<th>( \rho_{13}(q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>( \rho_{23}(q) )</td>
</tr>
<tr>
<td>( \rho_{23}(q) )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**SIMPLE EXAMPLE**
- \( \text{Det.M}_p = [1 - \rho^2] \)
- SMALLER THE \( \rho \), HIGHER THE \( \text{Det.M}_p \).
- \( \rho \) IS ESTIMATED BY BOUNDS.

**INFORMATION THEORY**
- \( H(X) \)
- \( H(Y) \)
- \( H(X,Y) \)

**INFORMATION THEORY**
- MUTUAL INFORMATION \( \text{MIF}(X,Y) \)
- \( H(X,Y) = H(X) + H(Y) - \text{MIF}(X,Y) \)
- IF \( \text{MIF}(X,Y) = 0 \),
- \( H(X,Y) = H(X) + H(Y) \)
- IF \( \text{MIF}(X,Y) > 0 \),
- \( H(X,Y) < H(X) + H(Y) \)

**MUTUAL INFORMATION**
- \( H(X,Y) = H(X) + H(Y) - \text{MIF}(X,Y) \)
- IF \( \text{MIF}(X,Y) = 0 \),
- \( H(X,Y) = H(X) + H(Y) \)
- IF \( \text{MIF}(X,Y) > 0 \),
- \( H(X,Y) < H(X) + H(Y) \)

**UNCERTAINTY REDUCTION AND MIF**
- \( H(\text{RANDOM}) = \)
- \( H(\text{SYSTEMATIC}) = \)
- \( H(\text{TOTAL}) \)
- WHEN MIF = 0
UNCERTAINTY REDUCTION AND MIF

- WHEN MIF > 0,
- $H(\text{TOTAL}) < H(\text{RAN}) + H(\text{SYS})$

MIF AND DET.$M_p$

- $\text{MIF}(X,Y) = \text{CONST. DET}.M_p$
- HIGHER THE VALUE OF DET.$M_p$,
  HIGHER THE MIF$(X,Y)$
- VALUE OF DET. $M_p$ IS MADE HIGHER BY USING
  EITHER UB OR LB FOR CORRELATED ELEMENTS.

APPLICATION I

- REDUCE OPTICAL MODEL DEFICIENCY
- NUCLEAR MODEL CALCULATIONS.
- UTILITY OF OPTICAL MODEL.
- OPTICAL MODEL PARAMETERS (OMP).
  $U + iW, R = R_oA^{1/3}, a$.
- OMP ARE HIGHLY CORRELATED.
  $^{239}\text{Pu}$ IN JENDL 3.2.

$^{239}\text{Pu}$ OMP CORR.MATRIX

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>R</th>
<th>W</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.95</td>
<td>1.0</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>R</td>
<td>0.99</td>
<td>-0.966</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>4.23</td>
<td>-0.558</td>
<td>0.492</td>
<td>1.0</td>
</tr>
<tr>
<td>a</td>
<td>4.92</td>
<td>-0.153</td>
<td>0.292</td>
<td>-0.294</td>
</tr>
</tbody>
</table>

VALUES OF BOUNDS AND THEIR DETERMINANT FOR OMP

<table>
<thead>
<tr>
<th>EX. VAL.</th>
<th>L. Bound</th>
<th>U. Bound</th>
<th>Det.LB</th>
<th>Det.UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.966</td>
<td>-0.989</td>
<td>0.137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.558</td>
<td>-0.7</td>
<td>-0.373</td>
<td>-0.574</td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.024)</td>
</tr>
<tr>
<td>-0.153</td>
<td>-0.429</td>
<td>-0.038</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

APPLICATION I

- SIMULATION OF OPTICAL MODEL PARAMETERS WITH REDUCED MODEL DEFICIENCY BY D-OPTIMAL CRITERION.
APPLICATION II

- GENERATION OF ROBUST RESONANCE PARAMETERS (RP).
- AVERAGED CROSS-SECTION.
- HIGHLY CORRELATED.
- PITFALLS IN SAMMY (ORNL) AND IN KALMAN (KYUSHU UNIVERSITY).

KALMAN FOR JENDL 3.2

- \( P = X - X C^T \{ C X C^T + V \}^{-1} C X \)
- \( P = \) FINAL COV.MATRIX OF RP.
- \( X = \) INITIAL COV.MATRIX OF RP.
- \( C = \) SEN.MATRIX.
- \( V = \) COV.MATRIX OF AVG.X-SEC.

CORR.MATRIX FOR \(^{235}\text{U}\) AVERAGED X-SEC.

<table>
<thead>
<tr>
<th>Energy Range (KeV)</th>
<th>Uncert. (%)</th>
<th>Exis.Val</th>
<th>U.Bound</th>
<th>L.Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>600-700</td>
<td>1.26</td>
<td>0.941</td>
<td>0.999</td>
<td>0.820</td>
</tr>
<tr>
<td>700-800</td>
<td>1.26</td>
<td>0.937</td>
<td>0.999</td>
<td>0.815</td>
</tr>
<tr>
<td>800-900</td>
<td>1.28</td>
<td>0.924</td>
<td>0.997</td>
<td>0.798</td>
</tr>
</tbody>
</table>

APPLICATION II

- AN INFORMATION THEORY APPROACH TO MINIMIZE CORRELATED SYSTEMATIC UNCERTAINTY IN MODELLING RESONANCE PARAMETERS.

APPLICATION III

- MAXIMIZATION OF REPRESENTATIVITY FACTORS (RF)
- COMPARISON OF NUCLEAR SYSTEMS.
- \( \delta R^2_{\text{new}} = \delta R^2_{\text{old}} \{1 - RF^2\} \)
- WHEN RF = 0, THEN, \( \delta R^2_{\text{new}} = \delta R^2_{\text{old}} \)
- WHEN RF > 0, THEN \( \delta R^2_{\text{new}} < \delta R^2_{\text{old}} \)

CORR.MATRIX MINOR ACTINIDES

<table>
<thead>
<tr>
<th></th>
<th>(^{241}\text{Am})</th>
<th>(^{242}\text{Am})</th>
<th>(^{242}\text{Cm})</th>
<th>(^{244}\text{Cm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{241}\text{Am})</td>
<td>1.0</td>
<td>0.94</td>
<td>0.94</td>
<td>0.62</td>
</tr>
<tr>
<td>(^{242}\text{Am})</td>
<td>1.0</td>
<td>0.99</td>
<td></td>
<td>0.61</td>
</tr>
<tr>
<td>(^{242}\text{Cm})</td>
<td></td>
<td>1.0</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>(^{244}\text{Cm})</td>
<td></td>
<td></td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>
BOUNDS FOR MINOR ACTINIDES

<table>
<thead>
<tr>
<th>EXI.VAL.</th>
<th>UB</th>
<th>LB</th>
<th>Det.UB</th>
<th>DET.LB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.94</td>
<td>0.98</td>
<td>0.88</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>0.94</td>
<td>0.98</td>
<td>0.88</td>
<td></td>
<td>0.88</td>
</tr>
<tr>
<td>0.62</td>
<td>0.84</td>
<td>0.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

UNCERTAINTIES IN X-SECTION MEASUREMENT

<table>
<thead>
<tr>
<th>σ</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Corr.Cff</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.5</td>
<td>1.0</td>
<td>0.3</td>
<td>p_{12} = 0.8</td>
</tr>
<tr>
<td>ε</td>
<td>1.6</td>
<td>2.2</td>
<td>1.3</td>
<td>p_{13} = 0.5</td>
</tr>
<tr>
<td>B</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>p_{23} = 0.9</td>
</tr>
<tr>
<td>p</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COV. AND COR. MATRIX

- COV.M M_{g}
- COR.M ρ
- RATIO OF X-SEC.
- WITHOUT COVARIANCE =4.20%
- WITH
- COVARIANCE=2%

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>EXISTING VALUE</th>
<th>LOWER BOUND VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ_{12}</td>
<td>0.77</td>
<td>0.32</td>
</tr>
<tr>
<td>ρ_{13}</td>
<td>0.80</td>
<td>0.37</td>
</tr>
<tr>
<td>ρ_{23}</td>
<td>0.87</td>
<td>0.16</td>
</tr>
</tbody>
</table>

APPLICATION III

- MAXIMIZATION OF REPRESENTATIVITY FACTORS FOR EXPERIMENTAL PLANNING OF CROSS-SECTION MEASUREMENTS.

APPLICATION IV

- TRANSMISSION MEASUREMENT OF IRON.
- \[ I = I_0 e^{-\sigma x} \]
- \( I_0 \) INITIAL INTENSITY.
- \( I \) FINAL INTENSITY AFTER TRANSMISSION I
- THICKNESS x.
- \( \sigma = \frac{1}{N_x} \log_2 \left( \frac{I_0}{I} \right) \)
- DEPENDS ON REDUCTION PARAMETERS.
- NEUTRON COUNTS WITH AND WITHOUT SAMPLE, BACKGROUND, DEAD TIME, etc.
APPLICATION IV

<table>
<thead>
<tr>
<th>ENERGY (MeV)</th>
<th>CROSS-SECTION. (Barns)</th>
<th>ORIGINAL SYS.UNCY (%)</th>
<th>SYS.UNCY BY MIF. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4-3.0</td>
<td>3.424</td>
<td>0.018</td>
<td>0.011</td>
</tr>
<tr>
<td>3.0-4.5</td>
<td>3.531</td>
<td>0.018</td>
<td>0.014</td>
</tr>
<tr>
<td>4.5-8.0</td>
<td>3.586</td>
<td>0.017</td>
<td>0.014</td>
</tr>
</tbody>
</table>

APPLICATION IV

• REDUCTION OF SYSTEMATIC UNCERTAINTY IN TRANSMISSION MEASUREMENT OF IRON BY ENTROPY BASED MUTUAL INFORMATION.
• P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, RADIATION MEASUREMENTS, 2009 (IN PRINT).

APPLICATION V

• REDUCTION OF SYSTEMATIC UNCERTAINTY IN RADIOPHARMACEUTICAL ACTIVITY.
• NEUTRON GENERATORS.
• ACTIVATION ANALYSIS.
• $^{99m}$Tc, $^{113m}$In.
• UNCERTAINTY IN FORMATION.

APPLICATION V

• $^{252}$Cf.
• $^{113}$In(n,n')$^{113m}$In.
• $C = N \sigma \phi \prod SYS.UNCY$.
• SYS.UNCY INTRODUCE NOISE.
• MINIMIZE THE NOISE BY REDUCTION OF SYSTEMATIC UNCERTAINTY.

APPLICATION V

• REDUCTION OF SYSTEMATIC UNCERTAINTY IN RADIOPHARMACEUTICAL ACTIVITY BY ENTROPY BASED MUTUAL INFORMATION.
• P.T.KRISHNA KUMAR AND HIROSHI SEKIMOTO, ITSURO KIMURA.
• TO APPEAR IN NUCLEAR INSTRUMENTS AND METHODS IN PHYSICS RESEARCH.

APPLICATION VI

• DECIPHERING ROBUST REACTOR KINETIC DATA.
• MEASUREMENT OF KINETIC PARAMETERS FOR AGCR.
• 665 Mwe, UO$_2$, 2.5% ENRICHED,
• CO$_2$, GRAPHITE MODERATOR.
• PITFALLS IN CHAUVENT'S CRITERION.
• REJECT DATA WITH HIGH CORRELATION COEFFICIENT.
## CORRELATION MATRIX

<table>
<thead>
<tr>
<th>COEFF.</th>
<th>FTC</th>
<th>HTFC</th>
<th>HTMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTC</td>
<td>1.0</td>
<td>0.849</td>
<td>0.373</td>
</tr>
<tr>
<td>HTFC</td>
<td>1.0</td>
<td>0.754</td>
<td></td>
</tr>
<tr>
<td>HTMC</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## CORRELATION COEFFICIENTS

<table>
<thead>
<tr>
<th>EXISTING VALUE</th>
<th>LOWER BOUND VALUES</th>
<th>DET.Mp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.849</td>
<td>-0.328</td>
<td></td>
</tr>
<tr>
<td>0.373</td>
<td>0.293</td>
<td>0.8097</td>
</tr>
<tr>
<td>-0.754</td>
<td>-0.174</td>
<td>0.0491</td>
</tr>
</tbody>
</table>

## APPLICATION VI

- Deciphering robust reactor kinetic data using mutual information.

## APPLICATION VII

- Classification of radioactive ores.
- Aerial survey.
- Mobile counting using NaI(Tl) detectors.
- Similarity measure.
- Higher correlation coefficient.

## APPLICATION VII

- Classification of radio elements using mutual information: A tool for geological mapping.

## APPLICATION VIII

- Design of sensors.
- Design of discriminating taste sensors using mutual information.
- Design of drugs.
- Salt (NaCl), Sour (HCL), Bitter (Quinine), Sweet (Sucrose), Umami (MSG).
ADVANTAGES OF MIF

• IMPROVE EXISTING VALUES.
• IMPROVE SYSTEMATIC UNCERTAINTY (CORRELATED).
• NO ASSUMPTIONS ABOUT DISTRIBUTION OF THE DATA.
• DISCRIMINATE STATISTICAL AND SYSTEMATICAL.

ADVANTAGES OF MIF

• ELEMENT WISE PROCESSING.
• ENTIRE STRUCTURE OF THE COVARIANCE MATRIX.
• AID EXPERIMENTALISTS TO IMPROVE METHOD OF MEASUREMENT AND INSTRUMENTATION.
• ROBUST AND FAST PROCEDURE.

• THANK YOU.